MODELING FLOW OF A VISCOUS LIQUID OVER A BLUNT PLANE PLATE

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A calculation technique and results are presented for characteristics of detached flow on the lateral surface of a blunted planar plate, based on the Navier–Stokes equation and a two-parameter turbulence model.

The plane plate is the simplest possible extended heat exchange surface. The presence of a blunt leading edge on the plate usually generates a detached flow. The hydrodynamic characteristics of such flows are fundamental in analyzing heat exchange processes.

Study of flow detachment, one of the most important problems of viscous flows, is of great significance in theoretical studies and practical applications. A broad range of material on detached flows has been collected in Chzhen's monograph [1]. In many cases detachment occurs as the result of interaction of the boundary layer with a positive pressure gradient along the flow over the body [1, 2]. The experimental studies [1, 3-6] have been dedicated to investigation of the detached flow which develops on the leading edge of blunt bodies. Theoretical studies of this problem involve great difficulties because of the inapplicability of the boundary layer equations in the given case: the boundary layer ahead of the flow detachment point is very thin, affecting the detachment process only weakly, while, a more significant role is played by the potential flow [6]. To describe the recirculation flow mathematically recourse is had to the Navier–Stokes equation [7]. In cases where turbulence of the incident laminar flow is considered and at high Reynolds number ($Re \geqslant 10^4$), where detachment and reattachment of the flow encourage transition into the turbulent regime, one must introduce a turbulence model into the system of equations, i.e., utilize equations describing turbulent flow.

Using the method of [7] the present study will correct the turbulence model and offer some results of a calculation of averaged characteristics of the processes considered.

Fundamental Equations. The flow of a viscous liquid can be described by the laws of conservation of mass, longitudinal and transverse momentum components, turbulent kinetic energy k, and the rate of dissipation of that quantity ε, written in differential form [7]:

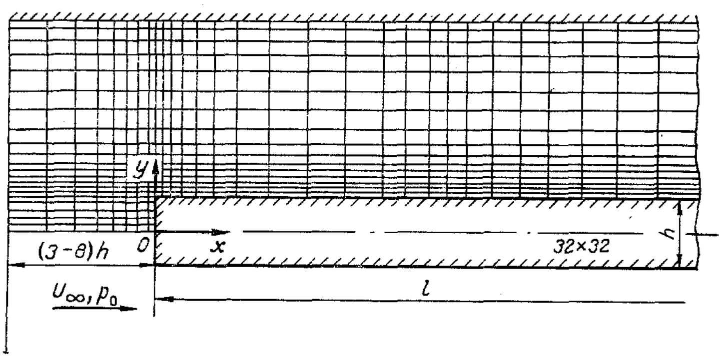

Fig. 1. Integration range with nonuniform difference grid.

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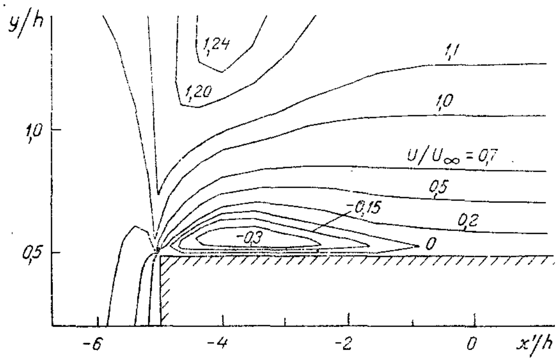

Fig. 2. Component velocity U/U_∞ isolines for flow over blunted plate by air: Re = $3\cdot 10^4$, Tu = 0.4%, Λ = 0.1, k_q = 0.05, x' = x − 5h, h = 0.0255 m, ℓ = 0.255 m.

$$\frac{\partial}{\partial x}(\rho U\Phi)+\frac{\partial}{\partial y}(\rho V\Phi)=\frac{\partial}{\partial x}\left(\Gamma_\Phi\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma_\Phi\frac{\partial \Phi}{\partial y}\right)+S_\Phi, \tag{1}$$

where Φ is the generalized dependent variable (Φ = U, V, k, ε); S_Φ is the source or drain for the variable Φ (Table 1).

The quantities G, μ_1, μ_t, f_μ, f_2 and R_t can be expressed by the following equations

$$G=\mu_1\left\{2\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial y}\right)^2\right]+\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)^2\right\}, \tag{2}$$

$$\mu_1=\mu+\mu_t, \tag{3}$$

$$\mu_t=C_\mu f_\mu \rho k^2/\varepsilon, \tag{4}$$

$$f_\mu=1-0{,}7\exp(-R_t/20), \tag{5}$$

$$f_2=1-0{,}22\exp[-(R_t/6)^2], \tag{6}$$

$$R_t=\rho k^2/(\varepsilon\mu). \tag{7}$$

The constants of the k—ε model are well known [7]: C_1 = 1.44, C_2 = 1.92, C_μ = 0.09, σ_ε = 1.3.

In the case of flow over a body in a channel boundary conditions must be determined on the input and output sections, on the channel wall, and on the body surface. For symmetric flow over a plate (see Fig. 1) we may restrict ourselves to determination of the boundary conditions in the upper portion of the channel alone. In such a case boundary conditions on the longitudinal axis of symmetry of the channel are also required.

Conditions at the channel input section are

TABLE 1. Values of Φ, Γ_Φ and S_Φ in Eq. (1)

Φ	Γ_Φ	S_Φ
Mass 1	0	0
Quantity of motion U	μ_1	$\frac{\partial}{\partial x}\left(\mu_1\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu_1\frac{\partial V}{\partial x}\right)-\frac{\partial p}{\partial x}$
Quantity of motion V	μ_1	$\frac{\partial}{\partial x}\left(\mu_1\frac{\partial U}{\partial y}\right)+\frac{\partial}{\partial y}\left(\mu_1\frac{\partial V}{\partial y}\right)-\frac{\partial p}{\partial y}$
Turbulent energy k	μ_1	$G-\rho\varepsilon-2\mu(\partial k^{1/2}/\partial y)^2$
Dissipation rate ε	$\mu+\frac{\mu_t}{\sigma_\varepsilon}$	$\frac{\varepsilon}{k}(C_1G-f_2C_2\rho\varepsilon)+2\frac{\mu\mu_t}{\rho}\left(\frac{\partial^2 U}{\partial y^2}\right)^2$

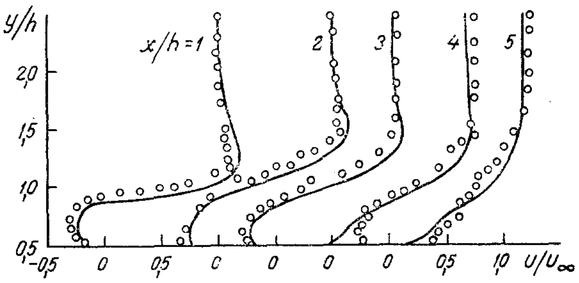

Fig. 3. Velocity profiles U/U_∞ in the detached flow zone (notation as in Fig. 2): points, experiment [3]; curves, present calculation.

$$U = U_\infty,\ V = 0,\ k = \left(\frac{\mathrm{Tu}\, U_\infty}{100}\right)^2,\ \varepsilon = \frac{(\mathrm{Tu}\, U_\infty)^3}{100^3 \Lambda h_1}; \tag{8}$$

at the output

$$V = \frac{\partial k}{\partial x} = \frac{\partial \varepsilon}{\partial x} = 0; \tag{9}$$

on the axis of symmetry

$$\frac{\partial U}{\partial y} = V = \frac{\partial k}{\partial y} = \frac{\partial \varepsilon}{\partial y} = 0; \tag{10}$$

and on the channel walls and plate surface

$$U = V = k = \varepsilon = 0. \tag{11}$$

System (1) can be solved by the finite difference method. The flow region is covered by a rectangular grid [7]. All variables except velocity are calculated at the grid points, a shifted grid being used for the velocity calculations [7]. The shift is performed such that the points of the new grid at which the velocity is calculated are located at the midpoint of the distance between pressure points at which pressure controls velocity.

In obtaining the finite difference equations it is assumed that each variable is included within its own intrinsic control area or "cell" [7], over which the differential equation is integrated.

The Patankar—Spaulding method is used to derive an equation for calculation of the unknown pressure from the continuity and momentum equations [7].

The difference equations are solved by the iteration method with external and internal iterations. The external iteration includes the following steps. Solution of the quantity of motion equation using the initial pressure p^* yields a field of intermediate velocities U^* and V^*, which due to the low accuracy of the specified pressure value p^* do not satisfy the continuity equation. Therefore the pressure correction p' equation is solved and the required changes in the velocities U' and V' are determined. Then the equations for the remaining variables (k, ε) are solved. The external iteration is repeated until solutions are obtained which satisfy the specified accuracy level between subsequent iterations.

Internal iteration is performed with solution of the difference equations for the specific variable Φ. The solution is carried out in the form of a block of iterations with simultaneous calculation of all Φ along each line of the grid. Since complete convergence is not required, one to three applications of the block procedure are sufficient. The difference equation and a detailed description of the method are presented in [7].

Calculation Results and Analysis. Experimental data on the problem under consideration [1-4] have encouraged the appearance of the method of [7], development of the calculation method presented, and in particular, use of Eq. (5) for determination of f_μ, used previously in the boundary layer problem [2].

Figure 2 shows overall pattern of change of the longitudinal velocity component characteristic of Reynolds numbers from 10^3 to 10^5: the detachment zone length is equal to $\approx 4h$,

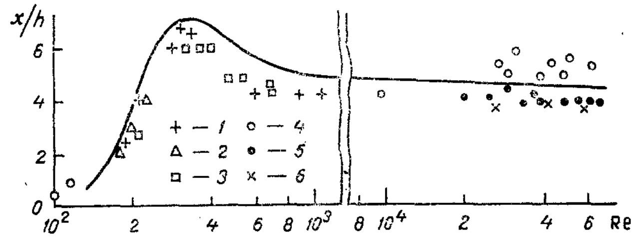

Fig. 4. Detachment zone length on lateral plate surface vs Re: 1-3) experimental data of [4], 4-6) [3], curve) present calculation, Tu = 0.4%, k_q = 0.05.

and the maximum reverse velocity is approximately 30% of the incident flow velocity, which agrees with the experimental data of [3]. Comparison of calculated and experimental [3] velocity profiles in the flow detachment zone (Fig. 3) shows that for numerical modeling the reverse zone thickness proves to be somewhat less than that given by the data of physical experiment. One cause of such divergence may be the fact that the mathematical model does not consider the effect of flow line curvature on the characteristics of flow over the plate considered.

It is well known [4] that solution of the Navier—Stokes equations show a dependence on Re of the reverse zone length, which in the laminar flow regime at small Re (100 < Re < 500) may extend from the leading edge of the plate a distance of 20 or more thicknesses. However, application of the k—ε model to this system of equations with increase in Reynolds number leads to a change in the calculation results. As the experimental data of [4] show, at $Re \approx 325$ the recirculation zone length is at a maximum $(L \approx 7h)$. In the detached shear layer transient velocity oscillations begin to appear, and with further increase in Re a transition to a turbulent flow regime occurs. Moreover the detachment zone becomes smaller, its length reaching its asymptotic value, equal to approximately four plate thicknesses. Therefore introduction of a turbulence model into the problem solution is necessary. However at Re < 325 the k—ε effect is insignificant and only the increased turbulence of the incident laminar flow need be considered.

Results of a calculation of detachment zone length (Fig. 4) obtained over a wide range of Reynolds number $(10^2 < Re \leqslant 10^6)$ agree with experimental data for both low [4] and high [3] Re.

The present mathematical model permits consideration of the effect of turbulence intensity and its scale on flow characteristics, which are specified by boundary condition (8). Table 2 presents results of a calculation showing the effect of Tu on detachment zone length and maximum reverse velocity therein. Increase in the turbulence scale Λ from 0.1 to 0.4 has a very slight effect on these characteristics.

Analysis of calculated and experimental [3] data on flow development downstream from the recombination point shows that the present method models the real overflow process with satisfactory accuracy. The experimental velocity profiles of [3] become similar in practice beginning at distances equal to 14 plate thicknesses. In calculating velocity profiles a higher value is obtained for this distance.

The calculated boundary layer form parameter H is also somewhat elevated as compared to experiment [3]. Hence it follows that the k—ε turbulence model generates reduced values of turbulence characteristics beyond the recombination point, and in this respect it must be refined in the direction of increasing pulsation characteristics downstream from the detached flow recombination point.

On the whole, the present method can be used to study detached flows developing as the result of interaction of a potential flow with a positive pressure gradient. It is evident from the results presented that the method is fully suitable for numerical modeling of average hydrodynamic characteristics of the detached zone which develops on the leading edge of a blunted plane plate.

TABLE 2. Detachment Zone Length and Maximum Reverse Velocity vs Intensity Turbulence, Re = 26900, k_q = 0.05, Λ = 0.1

Tu, %	0,4	1,0	5,0	10,0
L/h	3,5—4,0	3,3—3,8	1,8—2,3	0,9—1,5
U/U_∞	—0,30	—0,30	—0,27	—0,25

In performing the present calculations the ARTEK program package [8] for the BÉSM-6 computer was used, allowing modeling of the hydrodynamics and heat exchange of curvolinear bodies for planar and axisymmetric overflow situations.

NOTATION

h, plate height, m; h_1, channel height, m; H, form parameter, δ_1/δ_2; k, turbulent kinetic energy; k_q, channel encumbrance, h/h_1; L, detachment zone length, m; ℓ, plate length, m; U, V, average velocity components, m/sec; U_∞, incident flow velocity at channel input, m/sec; x, y, Cartesian coordinates, m; δ_1, displacement length, m; δ_2, momentum loss thickness, m; ε, turbulent energy dissipation; Λ, turbulence scale; μ, viscosity coefficient, $N \cdot sec/m^2$; ρ, density, kg/m^3; Re, Reynolds number $(U_\infty h\rho/\mu)$; Tu, flow turbulence $(100\sqrt{\overline{U^2}}/U_\infty, \%)$.

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